Rotation about an inclined axis: three dimensional matrices for reconstructing paleomagnetic and structural data

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Abstract—We derive matrices for calculating the three-dimensional finite rotation that restores a rigid block to its pre-displacement attitude. To use this technique an initial horizontal bedding is assumed, and the following orientations must be known: the expected field orientation of magnetization, the *in situ* orientation of the paleomagnetic vector and the bedding normal. The restoring rotation is decomposed into two convenient steps, represented by rotation matrices, such that they restore the *in situ* paleomagnetic vector together with the tilted bedding normal to their original orientation. These auxiliary rotations combine algebraically to a single finite rotation from which an inclined axis and amount of rotation are obtained. We demonstrate this method by reconstructing a distinct section of the Mount Sedom salt diapir (Dead Sea Rift Valley, Israel). Here, the conventional tilt correction that brings the bedding to horizontal fails to position the paleomagnetic vector in its expected orientation. Only a rotation of 125° about an axis which plunges 50° southward can restore both bedding and magnetization to their expected orientations.

INTRODUCTION

Tectonic rotation about horizontal or vertical axes is frequently observed in crustal deformation that involves folding and faulting (Ramsay 1967, Freund 1974, Garfunkel 1974, Ron et al. 1984, Kissel & Laji 1988). These two types of rotations are end-members of the more general rotation about an inclined axis (MacDonald 1980, Scotti et al. 1991). Rotation about an inclined axis occurs in complex fold structures and in oblique slip faulting (Ramsay 1961, 1967). Other processes that may be associated with rotation of rock masses about inclined axes are torsional motions on dipping fault planes (Mac-Donald 1980) and emplacement of coherent sedimentary slump blocks (Chan 1988). In such processes the conventional tilt correction of paleomagnetic data about a horizontal axis parallel to the strike of beds is not applicable, and leads to an artificial declination anomaly (MacDonald 1980).

MacDonald (1980) presented a stereographic construction to combine rotations and to obtain an inclined axis from it. Reidel et al. (1984) used this technique to analyze a large set of paleomagnetic data from a folded region in the Columbia Plateau, Washington. Chan (1988) noted that this technique involves laborious constructions with stereographic nets, and suggested a Declination Anomaly Chart (DAC) that enables numerous paleomagnetic and structural data to be plotted in an organized manner, and the angles of rotation for individual sites to be easily determined. The DAC mainly provides a means of detecting whether a set of anomalous paleomagnetic declinations may result from a rotation about an inclined axis. The plunge of the axis and the angle of rotation for an individual site can be determined from the DAC, but it does not provide the trend of the inclined axis. Another limitation of the DAC is that it requires angles of rotations not exceeding 90°.

This paper presents an algebraic method to calculate the orientation of an inclined axis for rotation of geological rock masses as well as the amount of rotation using structural and paleomagnetic measurements. The method is straightforward to program, it can handle a large volume of data, and it is applicable even for rotation exceeding 90°.

METHOD

A displacement history of a rigid body may comprise a sequence of finite rotations about different axes. However complicated that history may be, an equivalent rotation can restore the body from its final to its

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original position (e.g. LePichon *et al.* 1973). We are concerned here with identifying this equivalent rotation rather than determining the actual sequence of finite rotations that have displaced the rigid body. An equivalent rotation can be determined if two pairs of directions, before deformation and after it, are specified. In what follows, we focus on using the bedding pole and a paleomagnetic direction, before deformation and after it. We assume that the angle between the bedding pole and the paleomagnetic direction is conserved during the sequence of finite rotations. In a real case we anticipate that the magnitude of this angle before deformation would be somewhat different from its measured magnitude. This issue is addressed in some detail in the Discussion.

We choose a right-hand frame of reference (Fig. 1) and the right-hand convention (e.g. MacDonald 1980, Table 1) that implies a positive angle of rotation, ψ, when looking down the axis. First, the finite rotation is decomposed into two consecutive auxiliary rotations, represented by 3×3 transformation matrices. The first auxiliary rotation restores the tilted bedding normal $\mathbf{B}'$ to vertical $\mathbf{B}$ about a horizontal axis. The second auxiliary rotation restores the tilt corrected position of the measured (*in situ*) paleomagnetic vector $\mathbf{V}'$ to its expected orientation $\mathbf{V}$ about a vertical axis. These auxiliary rotations combine algebraically to a single rotation matrix from which the angle of rotation and the trend and plunge of an inclined axis is determined. To obtain the necessary data one should measure the trend and plunge of the dip direction, as well as isolate the *in situ* primary paleomagnetic vector $\mathbf{V}'$. We assume the bedding was initially horizontal ($\mathbf{B}$ is vertical), and we assess the expected orientation and polarity of the paleomagnetic vector $\mathbf{V}$ for the sample locality from coeval paleomagnetic poles for the plate on which the site resides. In such a case, the two pairs of vectors, bedding pole and paleomagnetic vector, are unequivocally determined (Fig. 2a). The two pairs of vectors, $\mathbf{V}-\mathbf{V}'$ and $\mathbf{B}-\mathbf{B}'$, convert from spherical to Cartesian components following Appendix A. Precaution must be taken for sedimentary rocks, where the inclination component of $\mathbf{V}$ may be shallower than expected due to sediment compaction (Arason & Levi 1990).

The combined effect of the two consecutive rotations, denoted R and P, is to bring the tilted bedding normal $\mathbf{B}'$ to vertical $\mathbf{B}$ and the measured (*in situ*) paleomagnetic vector $\mathbf{V}'$ to its expected orientation $\mathbf{V}$. The matrix elements of R (r_{ij}) and P (p_{ij}) can be calculated in several ways (LePichon *et al.* 1973, p. 37). We define the elements of the rotation matrices using the Cartesian coordinates of a rotation axis ($\mathbf{E} = [\mathbf{E}_x, \mathbf{E}_y, \mathbf{E}_z]$) and the angle rotation ψ (e.g. Cox & Hart 1986, p. 227; Appendix B). The auxiliary rotation R (Appendix B) restores the tilt by rotation about the strike of beds (Fig. 2b). This corrects the tilting (conventional 'tectonic' or 'stratigraphic' correction) and brings the bedding normal $\mathbf{B}'$ to the vertical $\mathbf{B}$ (Fig. 2b). For obtaining the positive axis of rotation in our right-handed frame of reference we add 90° to the dip azimuth. The rotation angle that restores
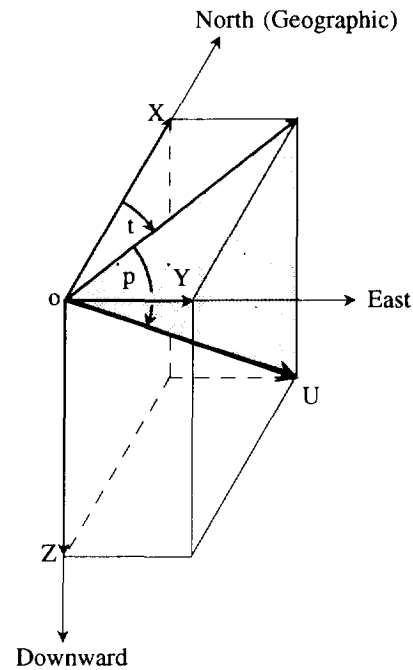

Fig. 1. A right-hand reference frame XYZ for orientation. The X-axis coincides with the geographic north, the Y-axis points eastward, and Z points downward. U is taken as a unit vector, the trend (t) of U is measured clockwise from the geographic north ($0° \leq t \leq 360°$), and plunge (p) of U is reckoned positive when the end of U points downward ($-90° \leq p \leq 90°$).

the tilt is given by the dip amount. The position of $\mathbf{V}''$, the tilt corrected position of the measured paleomagnetic vector $\mathbf{V}'$ after applying R, is given by its components

$$V_i'' = \sum_j (r_{ij} V_j').$$ (1)

Next, we apply the auxiliary rotation P (Appendix B), which is a rotation about a vertical axis (Fig. 2c), such that

$$V_i = \sum_j (p_{ij} V_j'').$$ (2)

The amount θ and sense of rotation P is given by the difference between the expected declination of $\mathbf{V}$ and the declination of $\mathbf{V}''$. The convenient way to calculate this azimuthal difference is to decompose $\mathbf{V}$ and $\mathbf{V}''$ to their spherical angles (Appendix A) and then to calculate the declination differences of these vectors. Notice that ideally $\mathbf{V}$ and $\mathbf{V}''$ would lie on a common horizontal small circle only if the inclination component of the primary paleomagnetic vector equals the inclination component of the expected paleomagnetic vector (see Discussion).

The equivalent rotation, T (t_{ij}), given by a matrix product of the auxiliary rotations, R and P, restores the bedding from its final (*in situ*) to its original horizontal position ($\mathbf{B}$ is vertical) and simultaneously restores the measured paleomagnetic vector to its expected orientation (Fig. 2d). Carrying out the matrix multiplication, we have
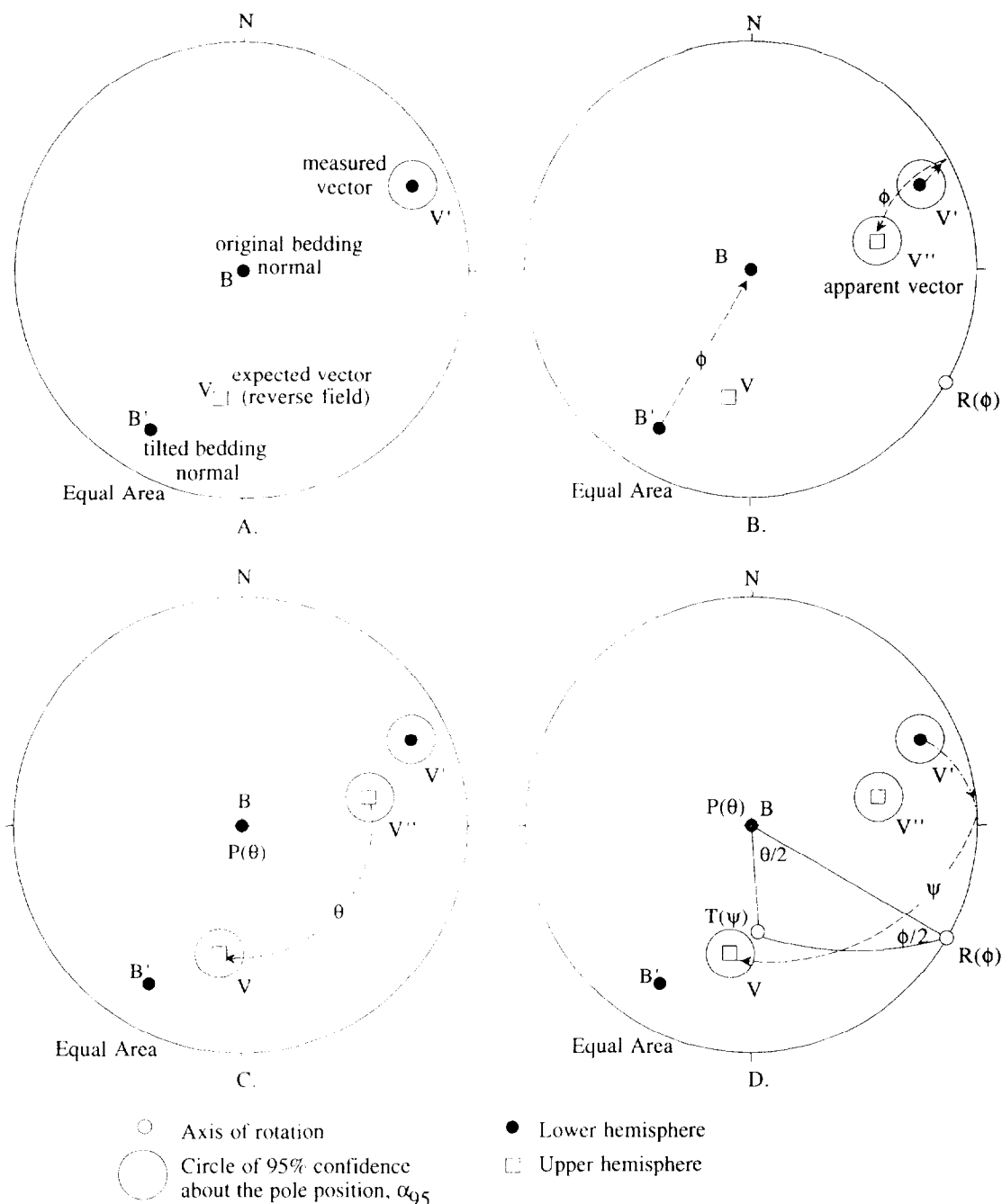
Fig. 2. Stereographic projections illustrating the stages of extracting an inclined axis from measured and expected bedding and paleomagnetic vectors. (A) Structural and paleomagnetic data. V' is the measured (in situ) paleomagnetic vector, V is the expected paleomagnetic vector (for reverse field). B' is the tilted bedding normal and B is the original bedding normal, assuming bedding has a zero initial dip. The value of α_{95} about V' is 8.2 and the value of α_{95} about V is 1.6°. (B) Auxiliary rotation R(φ): a rotation through the dip direction about an axis parallel to the strike of beds such that it corrects the tilting (conventional tectonic correction), and brings B' to B. V'' is an apparent position of the paleomagnetic vector. (C) Auxiliary rotation P(θ): a rotation about a vertical axis. The amount and sense of rotation P is given by θ, the difference between the original (expected) declination of V and the declination of V''. The restored V'' almost coincides with V. (D) The auxiliary rotations, R(φ) and P(θ), combine to a single (equivalent) rotation, T(η), and the inclined axis is obtained. The numerical data for this example are given in Table 1 and they are obtained from Mount Sedom case study.

$$t_{ij} = \sum_k (p_{ik} r_{kj}).\qquad(3)$$

The amount of rotation and the axis of rotation are directly obtained from the elements of T (Appendix C).

To summarize, we list the consecutive stages for obtaining a rotation axis and the angle of rotation using paleomagnetic and structural data. First, convert the spherical components of the two pairs of unit vectors

(Fig. 2a) to Cartesian components following Appendix A. Next, define the elements of the rotation matrix R [Appendix B and Fig. 2(b)]. Third, carry out equation (1) to find the apparent position of the paleomagnetic vector, V'' (Fig. 2b). Fourth, find the angle of rotation θ (Fig. 2c) which is the declination differences of V and V'' (Appendix A). Fifth, define the elements of the rotation matrix P (Appendix B and Fig. 2c). Next, define the elements of the rotation matrix T [equation (3)], which is the matrix multiplication of R by P

Table 1. Data for obtaining an inclined axis—the Mount Sedom example

Axis	Rotation axis Plunge	Trend	Rotation Angle	Pole	Bedding pole Plunge	Trend	Vector	Inc.	Dec.
				B' In situ	20°	210°	V' In situ	16°	63°
R(φ)	0°	120°	+70°	B" Apparent	90°	0°	V" Apparent	−42°	74°
P(θ)	90°	0°	+112°	B Original	90°	0°	V Expected	−39°	186°
T(ψ)	50°	175°	+125°						

A positive rotation is clockwise looking in the positive direction of the given axis. Inclination and declination of a paleomagnetic vector denoted Inc. and Dec., respectively. The close inclinations of V" and V support the assumption of a rigid body rotation.

Table 2. Numerical data for a test case

$$B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}; \qquad B' = \begin{pmatrix} -0.8138 \\ -0.4698 \\ 0.3420 \end{pmatrix};$$

$$V = \begin{pmatrix} -0.7728 \\ -0.0812 \\ -0.6293 \end{pmatrix}; \qquad V' = \begin{pmatrix} 0.4364 \\ 0.8565 \\ 0.2756 \end{pmatrix}; \qquad V'' = \begin{pmatrix} 0.2013 \\ 0.7208 \\ -0.6633 \end{pmatrix};$$

$$R = \begin{pmatrix} 0.5056 & -0.2849 & 0.8138 \\ -0.2849 & 0.8355 & 0.4698 \\ -0.8138 & -0.4698 & 0.3420 \end{pmatrix}; \qquad P = \begin{pmatrix} -0.3682 & -0.9297 & 0 \\ 0.9297 & -0.3682 & 0 \\ 0 & 0 & 1 \end{pmatrix};$$

$$T = \begin{pmatrix} 0.0784 & -0.6719 & -0.7365 \\ 0.5758 & -0.5726 & 0.5836 \\ -0.8138 & -0.4698 & 0.3420 \end{pmatrix}; \qquad \theta = 125.17° \\ \text{Axis} = 49.7°/175.8°$$

[Appendix B and Fig. 2(d)]. Finally, obtain the inclined axis and the angle of rotation following Appendix C (Fig. 2d).

APPLICATION

The Mount Sedom salt diapir, Dead Sea Rift Valley provides an excellent test case for applying the proposed technique, because the emergence of the diapir from the subsurface seemingly involves local rotation about inclined axes. In what follows we exemplify the method with a particular section of the Sedom sequence.

Mount Sedom is located on the southwestern side of the Dead Sea, in the Dead Sea Rift Valley, Israel. The mountain was formed by a penetrating salt body that has been rising since the Pleistocene (Zak 1967). The rocks of Mount Sedom consist mainly of rock salt, as well as layers of siltstone, dolomite, shale, and anhydrite of Neogene age. The dolomite and siltstone layers show brittle behavior as indicated by petrographic observations and undeformed fossils (Weinberger 1992). These layers show two components of magnetization. Field and laboratory tests led Weinberger (1992) to conclude that one of the components is a primary component that was acquired before the onset of diapirism, whereas the second component is a secondary CRM (chemical remanent magnetization) that was acquired after the emplacement of the diapir. We use the paleomagnetic and structural data from the most complicated part of the exposed diapir [the 'neck' region (Zak 1967)] to exemplify the method.

The attitude of beds, the orientation of the mean

primary vector, and the expected Neogene direction of magnetization for the dolomite and siltstone beds of the central Mount Sedom region are shown in Fig. 2(a) and Table 1.

Since simple rotation about a horizontal axis parallel to the bedding strike fails to align the primary paleomagnetic vector V' parallel to the expected orientation [Fig. 2(b), position V"], we reconstruct this locality by a single rotation about an inclined axis. Notice that although the polarity of the measured paleomagnetic vector (063°/16°) is positive, the rotation of this vector (together with the bedding) about an inclined axis consequently aligns the vector parallel to the reverse expected field (Fig. 2d). This is the case since the angle between V' and B' exceeds 90°. Table 1 summarizes the data needed to calculate the components of the inclined axis as well as the bedding and paleomagnetic orientation during stages of reconstruction. Table 2 gives the numerical data for obtaining the inclined axis, and may help debugging computer programs of the suggested method.

DISCUSSION

The suggested method can be considered as an application of a general algorithm. This algorithm gives a transformation that restores a rotated rigid plane if the rotations of a pair of directions in the plane are specified. The method may find additional application in structural geology, where structural elements are rotated relative to equivalent elements in adjacent domains. For example, opposing flanks of a fold can be

unfolded by matching respective pairs of foliations, lineations or paleomagnetic vectors. The application is also important in paleomagnetic reconstruction of faulted terrains, where slip is oblique and consequently rotation is about inclined axes such as in the Basin and Range (Ron 1993).

For comparison, we applied the alternative stereographic method suggested by MacDonald (1980, p. 3661) to the Sedom data (Table 1), and found that the inclined axis is 48°/175° and the angle of rotation is 127°. Hence, the solution of MacDonald yields only minor differences from our solution (Tables 1 and 2).

In the face of less than perfect data the reliability of the solution must be examined. First, the restriction of a rigid body rotation requires that after restoring the *in situ* bedding pole to vertical and the *in situ* declination to the expected, the inclination will be reasonably close to the expected value. If the values are not close the assumption of a rigid body rotation is violated, and the method is inapplicable. Checking this criterion in the Mount Sedom case study yields that the angle between the tilted bedding pole and the measured paleomagnetic vector is 132°, whereas the expected value is 129°. Hence, the assumption of a rigid body rotation seems relatively reasonable. Further reliability assessments are beyond the scope of this paper. However, we suggest here a straightforward prescription for estimating the confidence circle around the inclined axis. For this, one should calculate the confidence circle of a group of inclined axes, each of which restores a particular measured paleomagnetic site to the presumably well defined position of the expected paleomagnetic vector. This calculated confidence circle together with a scalar statistics for the angles of rotation give an easy-to-use criterion to the reliability of the solution.

In conclusion, we offer a convenient way to restore three-dimensional finite rotations using structural and paleomagnetic data. This method is straightforward, and gives a unique algebraic representation for the finite rotation of a rigid block.

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APPENDIX A

Converting spherical coordinates to Cartesian components and vice versa

The Cartesian reference frame together with the plunge (p) and trend (t) of a unit vector U are shown in Fig. 1. For convenience we limit the values to $-90° \leq p \leq 90°$. The spherical and Cartesian components are related by the equations:

$$U_x = \cos p \cos t; \tag{A1}$$

$$U_y = \cos p \sin t; \tag{A2}$$

$$U_z = \sin p. \tag{A3}$$

When converting from Cartesian coordinates to spherical angles, one first solves equation (A3) for p:

$$p = \arcsin U_z; \tag{A4}$$

If $p = 90°$ or $p = -90°$ then $t = 0°$

Next, if $U_y \geq 0$ then

$$t = \arccos [U_x/\cos p]; \tag{A5}$$

otherwise ($U_y < 0$),

$$t = 2\pi - \arccos [U_x/\cos p]. \tag{A6}$$

APPENDIX B

Define the elements of a rotation matrix

(1) S = s_{ij} has a general form of a rotation matrix. The elements of S are defined knowing the Cartesian components of a rotation axis, U = (U_x, U_y, U_z) (Appendix A), and an angle of rotation ψ (e.g. Cox & Hart 1986, p. 227; LePichon *et al.* 1973, p. 37). The elements are:

$$s_{11} = U_x U_x (1 - \cos \psi) + \cos \psi;$$

$$s_{12} = U_x U_y (1 - \cos \psi) - U_z \sin \psi;$$

$$s_{13} = U_x U_z (1 - \cos \psi) + U_y \sin \psi;$$

$$s_{21} = U_y U_x (1 - \cos \psi) + U_z \sin \psi;$$

$$s_{22} = U_y U_y (1 - \cos \psi) + \cos \psi;$$

$$s_{23} = U_y U_z (1 - \cos \psi) - U_x \sin \psi;$$

$$s_{31} = U_z U_x (1 - \cos \psi) - U_y \sin \psi;$$

$$s_{32} = U_z U_y (1 - \cos \psi) + U_x \sin \psi;$$

$$s_{33} = U_z U_z (1 - \cos \psi) + \cos \psi.$$

(2) Define the elements of the rotation matrix R: R (r_{ij}) restores the tilting about an axis parallel to the strike of beds. This axis is given by substituting $t = \tau + 90°$ (τ is the azimuth of the dip) and $p = 0°$ in equations (A1)–(A3), such that $U = (-\sin \tau, \cos \tau, 0)$. The amount of rotation is $\psi = \phi$, where ϕ is the dip angle (here taken positive for restoring the tilt).

$$r_{11} = \sin^2 \tau (1 - \cos \phi) + \cos \phi; \quad r_{12} = r_{21} = -\sin \tau \cos \tau (1 - \cos \phi);$$

$$r_{13} = -r_{31} = \cos \tau \sin \phi; \quad r_{22} = \cos^2 \tau (1 - \cos \phi) + \cos \phi;$$

$$r_{23} = -r_{32} = \sin \tau \sin \phi; \quad r_{33} = \cos \phi.$$

(3) Define the elements of the rotation matrix P: P (p_{ij}) is a rotation about a vertical axis. The magnitude and sense of the rotation P, denoted θ, is given by the declination difference between the original (expected) declination of V and the declination of V″. This axis is given by substituting $p = 90°$ in equations (A1)–(A3), such that $P = (0, 0, 1)$. The amount of rotations is $\psi = \theta$. The elements are:

$$p_{11} = p_{22} = \cos \theta; \quad p_{12} = -p_{21} = -\sin \theta;$$

$$p_{13} = p_{31} = p_{23} = p_{32} = 0; \quad p_{33} = 1.$$

APPENDIX C

Find an axis of rotation and an angle of rotation from a rotation matrix

The amount of rotation and the Cartesian components of an axis of rotation are calculated directly from the elements of a rotation matrix, s_{ij}, listed in Appendix B. The Eulerian elements are:

$$\psi = \arccos \left(\frac{s_{11} + s_{22} + s_{33} - 1}{2} \right) \tag{C1}$$

$$U_x = \frac{s_{32} - s_{23}}{2 \sin \psi} \tag{C2}$$

$$U_y = \frac{s_{13} - s_{31}}{2 \sin \psi} \tag{C3}$$

$$U_z = \frac{s_{21} - s_{12}}{2 \sin \psi}. \tag{C4}$$

One should obtain the spherical elements of the inclined axis following Appendix A.

Equation (C1) enforces the range of $0° \leq \psi \leq 180°$. This is convenient in that rotations are always positive. An inverse of a rotation is given by inverting the signs of the Cartesian components of U, which is equivalent to inverting the sign of ψ, or in general transposing s_{ij}.